

Hidden Variables, Statistical Mechanics and the Early Universe

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1 Introduction

One of the central mysteries of quantum theory is that it seems to be fundamentally nonlocal – and yet the nonlocality cannot be used for practical signalling at a distance. As argued elsewhere (Valentini 1991a,b; 1992; 1996), the consistency of modern physics seems to depend on a ‘conspiracy’, in which nonlocality is hidden by quantum noise. It is as if there is an underlying nonlocality which we are unable to control because of the statistical character of quantum events.

A natural explanation for this peculiar state of affairs is the hypothesis that quantum probability distributions are not fundamental, but merely represent a special state of equilibrium in which nonlocality happens to be masked by statistical noise (Valentini 1991a,b; 1992; 1996). In quantum theory a system with wavefunction ψ has an associated probability distribution given by the Born rule $\rho = |\psi|^2$. This is usually regarded as a fundamental law. If instead we regard $\rho = |\psi|^2$ as analogous to, say, the Maxwell distribution of molecular speeds for a classical gas in thermal equilibrium, the above ‘conspiracy’ takes on a different light: it is seen to be an accidental, contingent feature of the ‘quantum equilibrium’ distribution $\rho = |\psi|^2$. In a universe that is in (global) thermal equilibrium, it is impossible to convert heat into work; this is the classical thermodynamic heat death. In a universe that is everywhere in quantum equilibrium, it is impossible to use nonlocality for signalling; this is the ‘subquantum heat death’ which, we claim, has actually occurred in our universe.

The pilot-wave theory of de Broglie and Bohm (de Broglie 1928; Bohm 1952a,b) offers a concrete model of this scenario.¹ A system with wavefunction ψ is assumed to have a definite configuration $x(t)$ at all times whose velocity is determined by the de Broglie guidance equation $\pi = \partial S / \partial x$, where π is the canonical momentum and S is the phase of ψ (given locally by $S = \hbar \text{Im} \ln \psi$). The wavefunction ψ is interpreted as an objective ‘guiding field’ in configuration space, and satisfies the usual Schrödinger equation.²

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¹Note that in 1927 at the Fifth Solvay Congress de Broglie proposed the full pilot-wave dynamics in configuration space for a many-body system, not just the one-body theory; and unlike Bohm’s reformulation of 1952, de Broglie’s new approach to dynamics in the 1920s was always based on velocities (Valentini 2001a). The proceedings of the Fifth Solvay Congress are being translated into English (Bacciagaluppi and Valentini 2001).

²Pilot-wave dynamics may be applied to fields as well as particles – even to fermion fields (Valentini 1992; 1996; 2001a) and non-Abelian gauge theories (Valentini 2001a). Note that the natural spacetime structure associated with this velocity-based dynamics is Aristotelian spacetime $E \times E^3$, which has an inbuilt natural state of rest (Valentini 1997).

At the fundamental hidden-variable level, pilot-wave theory is nonlocal. For example, for two entangled particles A and B the wavefunction $\psi(x_A, x_B, t)$ has a non-separable phase $S(x_A, x_B, t)$ and the velocity $dx_A/dt = \nabla_A S(x_A, x_B, t)/m$ of particle A depends instantaneously on the position x_B of particle B.³ However at the *quantum* level, where one considers an ensemble with distribution $\rho(x_A, x_B, t) = |\psi(x_A, x_B, t)|^2$, operations at B have no statistical effect at A: as is well known, quantum entanglement cannot be used for signalling at a distance.

It is worth emphasising that this ‘washing out’ of nonlocality by statistical noise is peculiar to the distribution $\rho = |\psi|^2$. If one considers an ensemble of entangled particles at $t = 0$ with distribution $\rho_0(x_A, x_B) \neq |\psi_0(x_A, x_B)|^2$, it may be shown by explicit calculation that changing the Hamiltonian at B induces an instantaneous change in the marginal distribution $\rho_A(x_A, t) \equiv \int dx_B \rho(x_A, x_B, t)$ at A. For a specific example it was found that a sudden change $\hat{H}_B \rightarrow \hat{H}'_B$ at B – say a change in potential – leads after a short time ϵ to a change $\Delta\rho_A \equiv \rho_A(x_A, \epsilon) - \rho_A(x_A, 0)$ at A given by (Valentini 1991b)

$$\Delta\rho_A = -\frac{\epsilon^2}{4m} \frac{\partial}{\partial x_A} \left(a(x_A) \int dx_B b(x_B) \frac{\rho_0(x_A, x_B) - |\psi_0(x_A, x_B)|^2}{|\psi_0(x_A, x_B)|^2} \right) \quad (1)$$

(Here $a(x_A)$ depends on ψ_0 ; $b(x_B)$ also depends on \hat{H}'_B and is zero if $\hat{H}'_B = \hat{H}_B$.)

For nonequilibrium ensembles $\rho_0 \neq |\psi_0|^2$, there are genuine *instantaneous signals* at the statistical level. This has recently been shown to be a general feature, independent of pilot-wave theory. Any deterministic hidden-variables theory that reproduces quantum theory leads, for hypothetical nonequilibrium ensembles, to instantaneous signals at the statistical level (Valentini 2001b).

Note that our inability to see the trajectories – or hidden variables – is also a contingent feature of equilibrium: the uncertainty principle holds if and only if $\rho = |\psi|^2$ (Valentini 1991b).⁴ Heuristically, it is natural to compare our limitations with those of a Maxwell demon in thermal equilibrium with a gas, whose attempts to sort fast and slow molecules fail. The common objection to hidden variables – that their detailed behaviour can never be observed, making their existence doubtful – is seen to be misguided: for the theory cannot be blamed if we happen to live in a state of statistical equilibrium that masks the underlying details. There is no reason why nonequilibrium could not exist in the remote past or in distant regions of the universe (Valentini 1992), in which case the details of the ‘hidden-variable level’ would not be hidden at all.

Our central theme, then, is the subquantum ‘heat death of the universe’ at the hidden-variable level. How does one account for the quantum noise – encapsulated by the Born rule $\rho = |\psi|^2$ – that pervades our observed universe at the present time?

³It is being assumed here that the position of particle B could in some sense be changed while keeping the wavefunction fixed.

⁴Note, however, that the approximately classical functioning of the experimenter also plays a role here. A subquantum demon – an automaton that functions at the hidden-variable level – would see subquantum trajectories even for an equilibrium ensemble (Valentini 2001a).

2 ‘Empirical’ Approach to Statistical Mechanics

It will be shown how one may set up an analogue of classical statistical mechanics, based on pilot-wave dynamics in configuration space (Valentini 1991a; 1992; 2001a). We shall focus in particular on an analogue of the classical coarse-graining approach. But first, we shall address some foundational issues that were not considered in the author’s original papers.

In *what sense* might one ‘explain’ the probability distribution $\rho = |\psi|^2$, in the context of a deterministic (and time-reversal invariant) pilot-wave theory?

Say the Moon is now at position P (in phase space) at time t . This might be ‘explained’ – using Newton’s laws – by the fact that the Moon was (earlier) at position Q at time t_0 ; and if t_0 is in the remote past, one would in practice *deduce* that the Moon must have been at Q at time t_0 , from the observed position P today. That one has had to deduce the past (Q) from the present (P) would not change our physical intuition that the Moon may reasonably be said to be at P now ‘because’ it was at Q at t_0 .

This example may seem remote from statistical mechanics. However, many workers would object – in the author’s opinion wrongly – that Q at t_0 is a mere deduction, and *cannot* be regarded as a satisfactory explanation for P today. The argument would be that one would have a satisfactory explanation only if it could be shown that *all* – or at least ‘most’ – possible positions of the Moon at t_0 evolve into the Moon being at P today. Only then would one have an ‘explanation’ for the present, as opposed to a mere deduction about the past.

If this seems an unfair portrayal of what many workers in the foundations of statistical mechanics would claim, then consider what is attempted in that field. The ‘Holy Grail’ has always been to explain relaxation to thermal equilibrium by showing that all or ‘most’ initial states do so. And the great stumbling block has always been that, by time-reversal invariance, for every initial state that evolves towards equilibrium there is another that evolves away from it. One must then place restrictions on initial states, such as an absence of correlations, or a lack of fine-grained microstructure, and so on, restrictions that are violated by the unwanted ‘time-reversed’, entropy-decreasing states. One then tries to argue that the restricted set of initial states is ‘natural’, or that ‘most’ initial states – with respect to some suitable measure – satisfy the required conditions. As is well known, this program is as controversial today as it was a century ago.

To avoid such controversy one might adopt a more modest – and more realistic – ‘empirical’ approach (Valentini 1996; 2001a). Going back to the example of the Moon, one would try to deduce – or perhaps guess – where the Moon must have been before to explain its position today. In the same sense, for a box of gas evolving to thermal equilibrium, one should try to deduce or guess what the initial (micro-)state must have been like to yield the observed behaviour – without trying (in vain) to show that all or ‘most’ initial states would do so. Of course, in the case of a gas there will be a whole class of microstates yielding the required behaviour; one will not attempt to deduce the exact initial state uniquely (unlike in the case of the Moon). Further, because there are so many

variables one resorts to statistical methods. Thus, one tries to construct a class of initial conditions that yields the observed behaviour, and one tries to understand the evolution of that class towards a unique equilibrium state on the basis of a general mechanism (without having to solve the exact equations of motion).

In the case at hand, what is to be explained is the observation of equilibrium today to within a certain experimental accuracy, $\rho = |\psi|^2 \pm \epsilon$. For example, for a large number of independent Hydrogen atoms each with ground-state wavefunction ψ_{100} , one might measure the distribution of electron positions (with respect to the nuclei, in a nonrelativistic model) as accurately as current technology allows, and obtain a result $\rho = |\psi_{100}|^2 \pm \epsilon$. The initial or earlier conditions of the universe must have been such as to reproduce this result today.

One possible initial condition is just equilibrium itself (which is preserved by the equations of motion). That is, the universe could have started in equilibrium, leading to $\rho = |\psi|^2$ exactly today. But initial equilibrium is only one possibility among (uncountably) many: even without any proofs or assumptions about relaxation $\rho \rightarrow |\psi|^2$, it is clear on grounds of continuity alone that if initial equilibrium evolves to equilibrium today then an infinite class of initial states sufficiently close to equilibrium must evolve to $\rho = |\psi|^2 \pm \epsilon$ today; and given the violence of the early universe, one expects there to be a large class of initial states *far* from equilibrium that evolve to $\rho = |\psi|^2 \pm \epsilon$ today as well.

What sort of past conditions could have evolved to $\rho = |\psi|^2 \pm \epsilon$ today? Below, we shall hypothesise a class of possible initial (nonequilibrium) states and give a general mechanism explaining their approach to equilibrium, as is done in classical statistical mechanics.

It is not true, of course, that all initial states evolve towards equilibrium; time-reversal invariance forbids. Nor does it need to be true: ‘bad’ initial conditions are ruled out on *empirical*, observational grounds.

3 Subquantum *H*-Theorem

There are many approaches to classical statistical mechanics. Here we focus on a pilot-wave analogue of the classical coarse-graining *H*-theorem (Valentini 1991a). This is not because we believe coarse-graining to be the best approach; it has advantages and disadvantages, like the others.

For a classical isolated system, both the probability density p and the volume element $d\Omega$ (on *phase* space) are preserved along trajectories. The classical *H*-function $H_{class} = \int d\Omega p \ln p$ is constant in time. If we replace the fine-grained p by the coarse-grained \bar{p} and assume that $\bar{p}_0 = p_0$ at $t = 0$, then $\bar{H}_{class}(t) \leq \bar{H}_{class}(0)$ – the classical coarse-graining *H*-theorem of the Ehrenfests. The decrease of \bar{H}_{class} corresponds to the formation of structure in p and the consequent approach of \bar{p} to uniformity. (See for example Tolman (1938).)

Consider now an ensemble of complicated many-body systems, each with wavefunction Ψ , the configurations X distributed with a probability density P . The gradient ∇S of the phase of Ψ determines a velocity field \dot{X} in configuration space. (For a low-energy system of N particles of mass m , we have 3-vector

velocities $\dot{X}_i = \nabla_i S/m$ where $i = 1, 2, \dots, N$.) The continuity equations

$$\frac{\partial P}{\partial t} + \nabla \cdot (\dot{X} P) = 0 \quad (2)$$

(which holds by definition of P) and

$$\frac{\partial |\Psi|^2}{\partial t} + \nabla \cdot (\dot{X} |\Psi|^2) = 0 \quad (3)$$

(which follows from the Schrödinger equation) imply that the ratio $f \equiv P/|\Psi|^2$ is preserved along trajectories: $df/dt = 0$, where $d/dt = \partial/\partial t + \dot{X} \cdot \nabla$. Further, like $d\Omega$ classically, the element $|\Psi|^2 dX$ is preserved along trajectories too (where dX is the volume element in configuration space). This suggests replacing $p \rightarrow f$ and $d\Omega \rightarrow |\Psi|^2 dX$ in H_{class} , yielding the *subquantum H-function*

$$H = \int dX P \ln(P/|\Psi|^2) \quad (4)$$

This is in fact the relative negentropy of P with respect to $|\Psi|^2$.

The above continuity equations now imply that $H = H(t)$ is constant: $dH/dt = 0$, as in the classical case. But if one divides configuration space into cells of volume δV and averages P and $|\Psi|^2$ over the cells, yielding coarse-grained values \bar{P} and $\overline{|\Psi|^2}$, one may define a coarse-grained H -function

$$\bar{H} = \int dX \bar{P} \ln(\bar{P}/\overline{|\Psi|^2}) \quad (5)$$

If we assume the initial state has ‘no fine-grained microstructure’ at $t = 0$,

$$\bar{P}_0 = P_0, \quad \overline{|\Psi_0|^2} = |\Psi_0|^2 \quad (6)$$

then it may be shown that

$$\bar{H}_0 - \bar{H}(t) = \int dX |\Psi|^2 \left(f \ln(f/\tilde{f}) + \tilde{f} - f \right) \geq 0 \quad (7)$$

(where $\tilde{f} \equiv \bar{P}/\overline{|\Psi|^2}$), so that

$$\bar{H}(t) \leq \bar{H}_0 \quad (8)$$

for all t (Valentini 1991a).

It is instructive to examine the time derivatives of $\bar{H}(t)$ at $t = 0$. Use of the continuity equations shows that $(d\bar{H}/dt)_0 = 0$ and (Valentini 1992)

$$\left(\frac{d^2 \bar{H}}{dt^2} \right)_0 = - \int dX \frac{|\Psi_0|^2}{f_0} \left(\overline{(\dot{X}_0 \cdot \nabla f_0)^2} - \overline{(\dot{X}_0 \cdot \nabla f_0)}^2 \right) \leq 0 \quad (9)$$

The quantity in brackets is just the (non-negative) variance $\text{var}_{\delta V}(\dot{X}_0 \cdot \nabla f_0)$ of $(\dot{X}_0 \cdot \nabla f_0)$ over a coarse-graining cell δV . If we suppose that $\text{var}_{\delta V}(\dot{X}_0 \cdot \nabla f_0) \neq 0$

then we have the strict inequality $\bar{H}(t) < \bar{H}_0$ and $\bar{H}(t)$ *must decrease* over at least a finite time interval $(0, T)$ immediately after $t = 0$ (Valentini 2001a).

Because $x \ln(x/y) + y - x \geq 0$ for all real x, y , the coarse-grained H -function is bounded below by zero, $\bar{H} \geq 0$; further, $\bar{H} = 0$ if and only if $\bar{P} = \overline{|\Psi|^2}$ everywhere (Valentini 1991a). A decrease of \bar{H} towards its minimum value then corresponds to an approach of \bar{P} towards $\overline{|\Psi|^2}$.

The decrease of \bar{H} corresponds to a ‘stirring’ of the two ‘fluids’ P and $|\Psi|^2$ by the same velocity field \dot{X} , making P and $|\Psi|^2$ less distinguishable on a coarse-grained level. (This is similar to the classical Gibbs stirring of two liquids.) As in the corresponding classical case, the H -theorem gives us an insight into the mechanism whereby equilibrium is approached (for the particular class of initial conditions specified by (6)). Whether or not equilibrium is actually reached will depend on the system. It must be assumed that the initial steps towards equilibrium described by the H -theorem are actually completed in Nature, for appropriately complex systems, so that $\bar{H}(t) \rightarrow 0$ and $\bar{P} \rightarrow \overline{|\Psi|^2}$.

Given (coarse-grained) equilibrium $\bar{P} = \overline{|\Psi|^2}$ for our ensemble of many-body systems, it is straightforward to show that if a single particle is extracted from each system and prepared with wavefunction ψ , the resulting ensemble of particle positions has a (coarse-grained) distribution $\bar{\rho} = \overline{|\psi|^2}$ (Valentini 1991a). It is therefore to be envisaged that the equilibrium distribution seen today for ensembles of single particles arose via relaxation processes in the complex systems of which the particles were once part.⁵

4 An Estimate for the Relaxation Timescale

It is important first of all to have a rough, order-of-magnitude estimate for the timescale over which relaxation to quantum equilibrium takes place. What we have in mind here is not the time taken to reach equilibrium but the timescale over which there is a significant *approach* to equilibrium.⁶

We may define a relaxation timescale τ in terms of the rate of decrease of $\bar{H}(t)$ near $t = 0$. Because $(d\bar{H}/dt)_0$ vanishes one has to consider $(d^2\bar{H}/dt^2)_0$. Thus we define τ by $1/\tau^2 \equiv -(d^2\bar{H}/dt^2)_0/\bar{H}_0$ (Valentini 1992), where (9) gives $(d^2\bar{H}/dt^2)_0$ in terms of the initial state. Expanding $\dot{X}_0 \cdot \nabla f_0$ in a Taylor series within each coarse-graining cell of volume $\delta V = (\delta x)^{3N}$, it is found that $(d^2\bar{H}/dt^2)_0 = -I(\delta x)^2/12 + O((\delta x)^4)$ where $I \equiv \int dX (|\Psi_0|^2/f_0) |\nabla(\dot{X}_0 \cdot \nabla f_0)|^2$ (Valentini 2001a). Thus

$$\tau = \frac{1}{\delta x} \sqrt{\frac{12\bar{H}_0}{I}} + O(\delta x) \quad (10)$$

For δx small compared to the lengthscale over which $\dot{X}_0 \cdot \nabla f_0$ varies, $\tau \propto 1/\delta x$.

⁵For further details see Valentini (1991a,b; 1992; 2001a).

⁶Cf. the scattering time of classical kinetic theory.

Taking $\bar{H}_0 \sim 1$ (a mild disequilibrium) and crudely estimating I one obtains

$$\tau \sim \frac{1}{\delta x} \frac{m\hbar^2}{(\Delta P_0)^3} \quad (11)$$

where ΔP_0 is the (quantum) momentum spread of Ψ_0 (Valentini 2001a). Given $\tau \propto 1/\delta x$ this formula may also be obtained on dimensional grounds. Note that this result is merely a crude estimate.

5 Numerical Simulations

A numerical simulation of relaxation has been performed for the simplest possible case of an ensemble of independent particles in a one-dimensional box (Valentini 1992). The results were surprisingly good, given that the particles cannot move past each other. This simple model is of course an unrealistic setting for relaxation: one should really consider an ensemble of complicated systems with many degrees of freedom. Nevertheless the model is instructive.

Consider, then, a one-dimensional box with infinite barriers at $x = 0, L$ and energy eigenfunctions $\phi_n(x) = \sqrt{2/L} \sin(n\pi x/L)$ ($n = 1, 2, 3, \dots$) with eigenvalues $E_n = \frac{1}{2}(\pi n/L)^2$ (units $m = \hbar = 1$). The box contains an ensemble of independent particles, each guided by the same wavefunction $\psi(x, t)$. At $t = 0$ the wavefunction was taken to be a superposition of the first M eigenfunctions, with amplitudes of equal modulus but randomly-chosen phases θ_n :

$$\psi_0(x) = \sum_{n=1}^M \frac{1}{\sqrt{M}} \phi_n(x) \exp(i\theta_n)$$

The particles were taken to be uniformly distributed at $t = 0$: thus, $\rho_0(x) = 1/L$ and of course $\rho_0(x) \neq |\psi_0(x)|^2$. These initial conditions are sketched in Fig. 1, for the case $M = 10$ and $L = 100$.

By numerical calculation of the trajectories (given by $\dot{x} = (\partial/\partial x) \text{Im} \ln \psi$), the distribution $\rho(x, t)$ at later times may be determined, while the Schrödinger evolution of $\psi(x, t)$ is just

$$\psi(x, t) = \sum_{n=1}^M \frac{1}{\sqrt{M}} \phi_n(x) \exp i(\theta_n - E_n t)$$

For $L = 100$, $\psi(x, t)$ is periodic in time with period $T_P = 2\pi/E_1 \approx 12,700$ in our units. Because the particles cannot move past each other, each trajectory must recur with period T_P (to ensure that the equilibrium distribution recurs) and so any $\rho_0(x) \neq |\psi_0(x)|^2$ recurs as well. While this simple model exhibits a strong form of recurrence, nevertheless for times smaller than the recurrence time the approach to equilibrium is significant.

An example is shown in Fig. 2, again for ten modes ($M = 10$). At $t = 120$, ρ has developed sharp peaks which coincide with the (smooth) maxima of $|\psi|^2$.

An experimenter with a ‘blunt’ measuring device, say with resolution $\delta x = 10$, would conclude that $\bar{\rho}$ and $|\bar{\psi}|^2$ are roughly equal. The approach to equilibrium (at $t \ll T_P$) is surprisingly good for such a simple, highly constrained model.⁷

The evolution of $\bar{H}(t)$ has also been calculated (with C. Dewdney and B. Schwenker). An example is shown in Fig. 3, once again for $M = 10$, where \bar{H} is defined with a coarse-graining length $\delta x = 1$. The curve $\bar{H} = \bar{H}(t)$ shows a strict decrease soon after $t = 0$ (as it must); thereafter it decreases steadily but not monotonically, for $t \ll T_P$. (Of course, as the recurrence time is approached, \bar{H} necessarily increases as the initial conditions are restored.)

It is clear from Fig. 3 that \bar{H} initially decreases on a timescale of order ~ 100 . This result compares well with our crude estimate (11). For here $\Delta P_0 \approx (\pi/\sqrt{3})M/L$ and with $L = 100$ and $\delta x = 1$ our prediction is $\tau \sim 2 \times 10^5/M^3$. For $M = 10$ we have $\tau \sim 200$ – in fair agreement with the numerical calculation.

The timescale over which \bar{H} decreases – which may be quantified in terms of the time $t_{5\%}$ taken for \bar{H} to decrease by 5% – has been calculated (with C. Dewdney and B. Schwenker) for different numbers of modes, $M = 10, 15, \dots, 40$ (with fixed $\delta x = 1$), and for different coarse-graining lengths ranging from $\delta x = 0.2$ up to $\delta x = 2$ (with fixed $M = 20$). The results agree quite well with our predictions that $t_{5\%} \propto 1/M^3$ and $t_{5\%} \propto 1/\delta x$ (Dewdney, Schwenker and Valentini 2001; Valentini 2001a).⁸

The above model serves an illustrative purpose only. The particles cannot move past each other, and the approach to equilibrium is limited. Higher-dimensional models should show a much better approach to equilibrium.

Calculations analogous to the above are currently being performed by Dewdney, Schwenker and Valentini for a two-dimensional box. Bearing in mind the generally chaotic nature of the trajectories for this system (Frisk 1997), an efficient relaxation to equilibrium is to be expected.

6 Comments on ‘Typicality’

A completely different approach to explaining quantum equilibrium is based on a notion of ‘typicality’ for the initial configuration X_0^{univ} of the whole universe (Dürr *et al.* 1992a,b). Here, if Ψ_0^{univ} is the initial wavefunction of the universe, $|\Psi_0^{univ}|^2$ is taken to be the ‘natural measure’ on the space of initial configurations. It is shown that quantum distributions for measurements on subsystems are obtained for ‘almost all’ X_0^{univ} , with respect to the measure $|\Psi_0^{univ}|^2$.

While Dürr *et al.* give a general proof, their approach may be illustrated by the case of a universe consisting of an ensemble of n independent subsystems

⁷For further details see Valentini (1992; 2001a). Note that here the quantum timescale for the evolution of $|\psi|^2$ is $\Delta t \equiv \hbar/\Delta E \sim 70$, where ΔE is the quantum energy spread.

⁸In the first case, upon averaging over many trials with different random initial phases, a plot of $\ln t_{5\%}$ versus $\ln M$ was found to have a slope of -2.6 , compared with the predicted value of -3 . In the second case, there are departures from linearity for $\delta x \gtrsim 0.5$ due to the large variation in the velocity field over some coarse-graining cells, the result $\tau \propto 1/\delta x$ being valid only for small δx .

(which could be complicated many-body systems, or perhaps just single particles), each with wavefunction $\psi_0(x)$. Writing $\Psi_0^{univ} = \psi_0(x_1)\psi_0(x_2)\dots\psi_0(x_n)$ and $X_0^{univ} = (x_1, x_2, x_3, \dots, x_n)$, a choice of X_0^{univ} determines – for large n – a distribution $\rho_0(x)$ which may or may not equal $|\psi_0(x)|^2$.

Now it is true that, with respect to the measure $|\Psi_0^{univ}|^2$, as $n \rightarrow \infty$ almost all configurations X_0^{univ} yield equilibrium $\rho_0 = |\psi_0|^2$ for the subsystems. It might then be argued that, as $n \rightarrow \infty$, disequilibrium configurations occupy a vanishingly small volume of configuration space and are therefore intrinsically unlikely. However, for the above case, with respect to the measure $|\Psi_0^{univ}|^4$ almost all configurations X_0^{univ} correspond to the *disequilibrium* distribution $\rho_0 = |\psi_0|^4$. This has led to charges of circularity: that an equilibrium probability density $|\Psi_0^{univ}|^2$ is in effect being assumed for X_0^{univ} ; that the approach amounts to inserting quantum noise into the initial conditions themselves (Valentini 1996).⁹

However, there is a certain rationale to Dürr *et al.*'s approach. It has been argued (S. Goldstein, private communication) that in statistical mechanics one always rules out unwanted initial conditions in some way, usually by deeming them exceptional (or ‘untypical’) with respect to a specified measure. This is perfectly true. But in the author’s opinion, a class of initial conditions should be selected on *empirical* grounds. Even today we know only that, for all subsystems probed so far, $\rho = |\psi|^2 \pm \epsilon$ where ϵ is some experimental accuracy. There is certainly *no* reliable empirical evidence that $\rho = |\psi|^2$ exactly – or even approximately – in the very early universe, near the big bang.¹⁰

Note also that pilot-wave theory in exact equilibrium may never be susceptible to an experimental test of the basic law of motion (the de Broglie guidance equation); details of the trajectories may well be forever hidden from us, unless the human brain has an unexpected sensitivity to hidden variables (Valentini 2001a). Restricting pilot-wave theory to quantum equilibrium is as artificial as it would be to restrict classical mechanics to thermal equilibrium; and a huge amount of potentially new physics is lost thereby.

Finally, Dürr *et al.* focus on initial configurations for the whole universe while in Sect. 2 we emphasised initial distributions for subsystems. One might think that the former description is more correct because in the early universe there are no independent subsystems (that is, subsystems with their own wavefunction). But even leaving aside the point that the asymptotic freedom of interactions at high energies allows one to treat particles at very early times as essentially free, the two modes of description are really interchangeable: even if there are no independent subsystems, one can imagine (theoretically) what

⁹Note that if the word ‘typicality’ is replaced by ‘probability’, the result of Dürr *et al.* (1992a,b) becomes equivalent to the ‘nesting’ property proved by Valentini (1991a), which states that an equilibrium probability for a many-body system implies equilibrium probabilities for extracted subsystems, as briefly discussed above.

¹⁰In fact, the horizon problem in cosmology (the uniformity of the microwave background over regions larger than the classical causal horizons of a Friedmann universe) suggests that nonequilibrium $\rho \neq |\psi|^2$ may have existed at early times, the resulting nonlocality playing a role in homogenising the early universe (Valentini 1991b; 1992; 2001a). Inflation attempts to remove the horizon problem by changing the early expansion; but as yet there is no satisfactory inflationary theory. Early quantum disequilibrium offers an alternative approach.

would happen if (say) single particles were extracted from a gas of interacting particles, prepared with a certain wavefunction ψ , and had their positions measured. Whether the measured distribution ρ is equal to $|\psi|^2$ or not will depend on the configuration of the whole system prior to the extraction. By this (purely theoretical) device, one could characterise the total configuration as ‘equilibrium’ or ‘nonequilibrium’ (Valentini 2001a). Our Sect. 2 could then be rephrased in terms of configurations rather than distributions: our ‘empirical’ approach would amount to trying to find out what sort of initial configuration could explain the statistics observed today – and again, while an initial ‘equilibrium’ configuration is a possibility, there are clearly many ‘nonequilibrium’ configurations at $t = 0$ that explain the observations just as well.

7 The Early Universe. Suppression of Relaxation at Early Times

At first sight one might think that, even if the universe did begin in quantum nonequilibrium, relaxation $\rho \rightarrow |\psi|^2$ would take place so quickly (in the extreme violence of early times) as to erase all record of nonequilibrium ever having existed. However, closer analysis reveals a plausible scenario whereby quantum nonequilibrium could survive to the present day, for some types of particle left over from the big bang, leading to observable violations of quantum theory.

The scenario has three stages: (i) depending on how the relaxation timescale τ scales with temperature T , it is possible that relaxation will be *suppressed* at very early times by the very rapid expansion of space, in which case (ii) those particles that decouple at very early times will still be out of quantum equilibrium at the time of decoupling, after which (iii) the free spreading of particle wavefunctions *stretches* any residual nonequilibrium up to larger lengthscales that are within reach of experimental test. This scenario will now be briefly sketched, in this section and the next.¹¹

In the standard hot big bang model, the early universe contains what is essentially an ideal gas of effectively massless (relativistic) particles. It is instructive to recall the standard reasoning about how thermal equilibrium between distinct particle species is achieved: the interaction timescale t_{int} – that is, the mean free time between collisions – must be smaller than the expansion timescale $t_{exp} \equiv a/\dot{a} \sim (1 \text{ sec})(1 \text{ MeV}/kT)^2$, where $a(t) \propto t^{1/2}$ is the scale factor and t is the time since the beginning.¹² As is well known, this condition need not always be satisfied: for instance, if the cross section $\sigma(T)$ falls off faster than $1/T$ at high temperatures then $t_{int} \sim 1/n\sigma$ – with a number density $n \sim T^3$ (in natural units where $\hbar = c = 1$) – will fall off slower than $1/T^2$ so that, at sufficiently high temperatures, $t_{int} \gtrsim t_{exp}$ and the particles do not thermalise. The functions $\sigma = \sigma(T)$ are determined by particle physics; once they are known, it is possible to determine the thermal history of the universe, and in particular

¹¹For further details see Valentini (2001a,c).

¹²See any standard cosmology text.

to deduce when particle species fall out of thermal contact with each other.

Similar reasoning applies to the relaxation $\rho \rightarrow |\psi|^2$. There will be a temperature-dependent timescale $\tau = \tau(T)$ over which relaxation takes place. There will also be a *competing* effect due to the expansion of space: as space expands wavefunctions are stretched, and it is easy to see in pilot-wave theory that this results in a proportionate expansion of the disequilibrium lengthscale.

To see this, consider an initial distribution $\rho_0(x)$ with disequilibrium on a small lengthscale δ_0 . We may write $\rho_0(x) = |\psi_0(x)|^2 f_0(x)$, where $\psi_0(x)$ has some width Δ_0 and $f_0(x) \neq 1$ varies over distances $\delta_0 \ll \Delta_0$ (so that coarse-graining over distances much larger than δ_0 yields an equilibrium distribution). If the wavefunction expands up to a width $\Delta(t)$, then because f is conserved along trajectories, and because particles initially separated by a distance δx_0 are later separated roughly by $\delta x(t) \sim (\Delta(t)/\Delta_0) \delta x_0$, it follows that deviations $f \neq 1$ occur on an expanded lengthscale $\delta(t) \sim (\Delta(t)/\Delta_0) \delta_0$. This is true irrespective of whether the wavefunction expands because of the expansion of space or simply because of its own natural free spreading over time.¹³

Thus relaxation $\rho \rightarrow |\psi|^2$ occurs if $\tau \lesssim t_{exp}$, while it is *suppressed* if $\tau \gtrsim t_{exp}$. The issue now depends on how τ scales with T . Unfortunately, there are as yet no reliable calculations that can tell us this. But we can make a crude estimate.

Given $\tau \propto 1/\delta x$, for massless particles τ may be estimated on purely dimensional grounds to be $\tau \sim (1/\delta x) \hbar^2/c(\Delta p)^2$ where Δp is the particle momentum spread. Taking $\Delta p \sim kT/c$ we have $\tau \sim (1/\delta x) \hbar^2 c/(kT)^2$ which is larger than $t_{exp} \sim (1 \text{ sec})(1 \text{ MeV}/kT)^2$ for all T if $\delta x \lesssim 10l_P$ (where l_P is the Planck length). However, it would be more realistic to apply our estimate for τ to a coarse-graining length δx that is relevant to the energetic processes at temperature T , the natural choice being the thermal de Broglie wavelength $\delta x \sim \hbar c/kT$ (the typical width of particle wavepackets). We then have $\tau \sim \hbar/kT$ – a plausible result, according to which relaxation is suppressed ($\tau \gtrsim t_{exp}$) if $kT \gtrsim 10^{18} \text{ GeV} \approx 0.1 kT_P$ or $t \lesssim 10t_P$ (where T_P and t_P are the Planck temperature and time). Our simple estimate suggests that relaxation to quantum equilibrium began one order of magnitude below the Planck temperature.

Note that in the standard Friedman expansion considered here, the rate $1/t_{exp}$ at which space is expanding becomes infinite as $t \rightarrow 0$. Of course, our relaxation rate $1/\tau \propto T$ also tends to infinity as $t \rightarrow 0$ (and $T \rightarrow \infty$), but this is offset by the expansion rate $1/t_{exp} \propto T^2$ which tends to infinity even faster. Note also that while our estimate is very crude and should not be taken too seriously, it does illustrate the key point that, if τ decreases with temperature more slowly than $1/T^2$, then relaxation will be suppressed at very early times.

It is therefore plausible that any particles that decouple soon after the Planck era will *not* have had time to reach quantum equilibrium, the extreme violence of that era being offset by the even more extreme expansion of space. And at the time of decoupling, such particles are expected to show deviations $\rho \neq |\psi|^2$ on a lengthscale $\delta x \sim \hbar c/kT_P = l_P \approx 10^{-33} \text{ cm}$.

¹³This is consistent with the H -theorem, for H measures negentropy relative to $|\psi|^2$: if $|\psi|^2$ itself expands, the absolute disequilibrium lengthscale can increase without increasing H .

8 Residual Disequilibrium Today. Experimental Tests

After decoupling, particle wavefunctions undergo a huge expansion – partly due to the expansion of space itself and partly due to the free spreading of the wavepacket through space. This results in a proportionate stretching of disequilibrium to much larger lengthscales, as shown above.¹⁴

For example, relic gravitons are believed to decouple at $kT_{dec} \sim 10^{19}$ GeV or at redshift $z_{dec} \sim T_{dec}/T_{now} \sim 10^{32}$. The subsequent expansion of space alone, by a linear factor of $\sim 10^{32}$, will stretch the disequilibrium lengthscale up to ~ 1 mm. Of course, there seems to be little hope of detecting the ~ 10 K graviton background directly in the near future – still less of testing it for violations of quantum theory. However, it is expected that there are other, more exotic particles that decoupled soon after T_P . (Supersymmetry and string theory predict a plethora of new particles at high energies.) These may be no easier to detect directly. However, some of them might decay at later times into more easily detectable particles such as photons, perhaps by annihilation $X + \bar{X} \rightarrow 2\gamma$ or by a supersymmetric decay $X \rightarrow \gamma + \tilde{\gamma}$ into a photon and a photino. Nonequilibrium for the parent particles should yield nonequilibrium for the decay products as well. Thus, we would suggest testing the Born rule $\rho = |\psi|^2$ for photons produced by the decay of exotic relic particles from the Planck era. For example, in a two-slit interference experiment, such photons might produce an interference pattern that deviates from the quantum prediction.

Experiments are under way searching for exotic relic particles supposed to make up the ‘dark matter’ pervading the universe. Some of these experiments involve searching for decay photons. However, the usual dark matter candidates (such as neutrinos, neutralinos, axions or gravitinos) are expected to decouple much later than t_P . Nevertheless, our knowledge of particle physics beyond the standard model is so uncertain that, for all we know, there might exist an appropriate relic particle X – that decouples soon after t_P and partially accounts for dark matter. If such particles were detected, they or their decay products would be our prime candidates for particles violating the Born rule.

In the meantime one might also consider the relic photons that make up the microwave background. But why consider particles that decoupled at $t \sim 10^5$ yr – *much* later than t_P – when our estimates suggest that any disequilibrium will have been erased? One answer is that the hugely expanded wavepackets of relic particles provide an interesting test of the Born rule in extreme conditions. On general grounds (irrespective of the suggestions made here about hidden variables and early nonequilibrium), it would be worth testing quantum theory in such unusual circumstances, where quantum probabilities have spread over

¹⁴Decoupling is of course never exact. Because of small residual interactions at all times, the wavefunctions of relic particles contain tiny scattering terms that perturb the trajectories – possibly ‘re-mixing’ the expanding disequilibrium. Calculation of the effect of scattering by a tenuous medium shows that re-mixing does *not* happen: the trajectory perturbations grow only as $t^{1/2}$ and cannot overcome the linear ($\propto t$) free expansion of the disequilibrium lengthscale (Valentini 2001a,c). We may therefore safely ignore such residual interactions.

intergalactic distances. Relic photons decouple at $kT_{dec} \sim 1 \text{ eV}$ ($z_{dec} \sim 10^3$) and at decoupling their wavepackets have widths of order $\sim \hbar c / (1 \text{ eV}) \sim 10^{-5} \text{ cm}$. If the packets have (as is widely assumed) spread essentially freely at the speed of light for $\sim 10^{10}$ years then today they will have a width $\sim 10^{28} \text{ cm}$ and so the packets will have expanded by $\sim 10^{33}$ (ignoring the relatively small effect of the expansion of space by a factor $\sim 10^3$). It can be argued – on general grounds, independent of pilot-wave theory – that for cosmological microwave photons the quantum probability today on lengthscales $\sim 1 \text{ cm}$ could contain traces of corrections to the Born rule which may have existed (for whatever reason) on the Planck scale at the time of decoupling (Valentini 2001a,c).¹⁵

As suggested elsewhere (Valentini 1996), it would then be worthwhile to test quantum theory for photons from the microwave background. In a two-slit interference experiment with single relic photons, the usual quantum interference pattern – given by $\rho = |\psi|^2$ – might show an *anomalous blurring*.¹⁶

Generally, corrections to the Born rule for any photons from deep space would produce a number of observable anomalies. In particular, the functioning of some astronomical instruments might be affected: diffraction-grating spectrometers could produce unreliable readings, and the diffraction-limited performance of some telescopes might be impaired.¹⁷

9 Outlook

It is clear that much remains to be done to develop the above ideas fully. Other approaches to subquantum statistical mechanics (based for example on external perturbations) remain to be developed. Properties of the trajectories such as ergodicity and mixing (in a rigorous sense) should be investigated.

The process of relaxation to quantum equilibrium should also be studied further, in particular in the early universe. It remains to be seen if the relaxation timescale τ really does decrease with temperature more slowly than $1/T^2$, so that relaxation is suppressed at very early times. An experimental test of the Born rule $\rho = |\psi|^2$ for relic cosmological particles seems feasible, in particular for photons from the microwave background (Valentini 2001a,c).

As for the issue of chance in physics, the central conclusion of this work is that, like the cosmic microwave background, *quantum noise is a remnant of the big bang*. And just as the microwave background has been found to have small nonuniformities in temperature, so the ‘quantum background’ – the quantum noise that pervades our universe – may have small deviations from the Born rule $\rho = |\psi|^2$, and should be probed experimentally.

¹⁵Our point here is that the line of argument given in this paper has led us to propose experimental tests that are actually worthwhile in their own right, because they would probe quantum theory in new and extreme conditions.

¹⁶Incoming photons will have very nearly plane wavepackets, so there is no ambiguity as to what quantum theory would predict in such an experiment. Note also that it has recently become possible to detect single photons in the far-infrared region (Komiya *et al.* 2000).

¹⁷For details see Valentini (2001a).

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FIGURE CAPTIONS

Fig. 1. Initial conditions at $t = 0$. We have plotted $|\psi|^2$ for a superposition of ten modes with amplitudes of equal modulus and random phase. The initial density ρ is taken to be uniform.

Fig. 2. At $t = 120$. There is a strong coincidence between the sharp peaks of ρ and the smooth peaks of $|\psi|^2$, indicating an approximate approach to equilibrium on a coarse-grained level.

Fig. 3. Plot of the coarse-grained \bar{H} -function against time.

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